Passive Convection of Density Fluctuations in the Local Interstellar Medium

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Abstract. We have developed a time-dependent three-dimensional model of isotropic, adiabatic, and compressible magnetohydrodynamic plasma to understand nonlinear cascades of density fluctuations in local interstellar medium. Our simulations, describing evolution of initial supersonic, super Alfvénic plasma modes, indicate that nonlinear interactions lead to damping of plasma motion. During the process, turbulent cascades are governed predominantly by the Alfvénic modes and velocity field fluctuations evolve towards a state charachterized by near incompressibility. Consequently, density field is advected passively by the velocity field. Our findings thus demonstrate that the observed density fluctuations in the interstellar medium are the structures passively convected by the background velocity field.

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INTRODUCTION

Density fluctuations in the local interstellar medium (ISM) are observed to follow a Kolmogorov-like $k^{-5/3}$ spectrum [1] that spans almost 12 decades in wavenumber space [2]. This enigmatic observation has remained a source of inconspicuous understanding of the origin, nature and dynamics of the ISM plasma fluctuations. Interestingly, the Kolmogorov-like $k^{-5/3}$ spectrum is a characteristic of fully developed incompressible hydrodynamic turbulence. Why compressible and magnetized ISM plasma behaves like an incompressible hydrodynamic fluid has been elusive and has certainly obscured our understanding of small-scale ISM turbulence. Small-scale turbulent ISM fluctuations are not only potentially important in the context of the global heliosphere, i.e. evolution of the termination shock and its interaction with local small-scale upstream/downstream turbulence, but are also instrumental to our understanding of many astrophysical phenomena including energization and transport of cosmic rays, gamma-ray bursts, ISM density spectra, etc. To understand the origin and dynamical evolution of turbulent density field in the local ISM, we in this paper present our results of a self-consistent ISM turbulence simulation model that we have devloped based on time-dependent, three-dimensional magnetohydrodynamic equations.

MHD MODEL

Our model assumes that the ISM turbulent fluctuations in the plasma are isotropic, homogeneous, thermally equilibrated and fully developed. Second, no mean magnetic field and velocity flows are present at the outset. There may however be local mean

flows generated by self-consistently excited nonlinear instabilities. We further assume that the characteristic turbulent correlation length-scales are typically much bigger the shock characteristic scale-lengths in the ISM flows. Additionally, underlying turbulent correlation length-scales are considered to be large enough to treat any localized shocks as smooth discontinuities. In other words, the characteristic shock length-scales are small compared to the ISM turbulent fluctuation length-scales, and finally boundary conditions are periodic, since we are not aware of any realistic boundary conditions for the present problem. Periodicity is thus a natural and most appropriate choice for modeling the local ISM. Statistically homogeneous, isotropic and isothermal MHD plasma can be cast in terms of a single fluid density $\rho(\mathbf{r},t)$, magnetic $\mathbf{B}(\mathbf{r},t)$ and velocity $\mathbf{U}(\mathbf{r},t)$ fields and pressure $p(\mathbf{r},t)$ as

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0, \tag{1}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}, \tag{2}$$

$$\rho \left(\frac{\partial}{\partial t} + \mathbf{U} \cdot \nabla \right) \mathbf{U} = -\nabla p + \frac{1}{4\pi} (\nabla \times \mathbf{B}) \times \mathbf{B} + \nu \nabla^2 \mathbf{U} + \hat{\eta} \nabla (\nabla \cdot \mathbf{U}). \tag{3}$$

$$\nabla \cdot \mathbf{B} = 0. \tag{4}$$

The equations are closed with an equation of state relating the perturbed density to the pressure variables. Here $\mathbf{r} = x\hat{\mathbf{e}}_x + y\hat{\mathbf{e}}_y + z\hat{\mathbf{e}}_z$ is a three dimensional vector, η and ν are, respectively, magnetic and kinetic viscosities. Note carefully that MHD plasma momentrum equation, i.e. Eq. (3), contains nonlinear dissipative terms on the right hand side (*rhs*). This means that dissipative processes can potentially be mediated by nonlinear interactions, in addition to the damping associated with the small-scale turbulent motion. Thus nonlinear turbulent cascades are not only responsible for the spectral transfer of energy in the inertial range, but also likely to damp the plasma motion in a complex manner. Nonetheless, the spatio-temporal scales in the nonlinear damping can be *distinct* from that of the linear dissipation. We will quantify the damping associated with the nonlinear interactions in the subsequent section.

The above equations can be normalized using a typical length scale (ℓ_0), density (ρ_0), pressure (p_0), magnetic field (B_0) and the velocity (U_0). With respect to these normalizing ambient quantities, one may define a constant sound speed $C_{s_\circ} = \sqrt{\gamma p_0/\rho_0}$, sonic Mach number $M_{s_\circ} = U_0/C_{s_\circ}$, Alfvén speed $V_{A_\circ} = B_0/\sqrt{4\pi\rho_0}$, and Alfvénic Mach number $M_{A_\circ} = U_0/V_{A_\circ}$. The magnetic and mechanical Reynolds numbers are $R_{m_\circ} \approx U_0\ell_0/\eta$ and $R_{e_\circ} \approx U_0\ell_0/\nu$, and the plasma beta $\beta_0 = 8\pi p_0/B_0^2$. While these quantities arise purely out of the normalizations and are associated with a bulk (large-scale) plasma motion, there can exist turbulent speeds, Mach and Reynolds numbers which depend locally on the small-scale and relatively high frequency fluctuations. This is illustrated schematically in Fig. (1). It is this component that describes the high frequency contribution corresponding to the acoustic time-scales in the modified pseudosound relationship proposed in the Nearly Incompressible (NI) theory by [3, 4, 5]. Moreover, this high frequency component is also related with the nonlinear damping of plasma

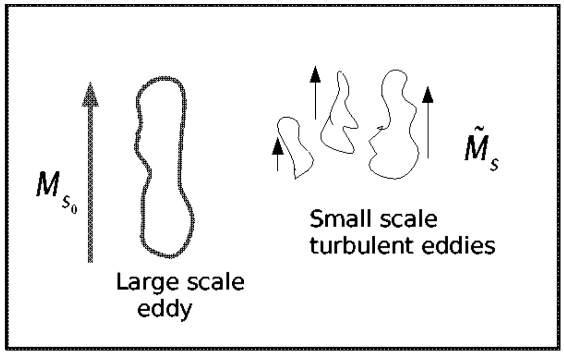

FIGURE 1. Schematic of the Mach number as determined from the large-scale flows (left) and small-scale fluctuations (right). A large-scale flow or constant mean background flow leads typically to a constant Mach number, whereas local fluctuating eddies give rise to turbulent Mach numbers which depend upon local properties of high frequency and smaller-scale turbulent fluctuations.

motion as described above. We define the sound speed excited by the small-scale turbulent motion as $\tilde{C}_s(\mathbf{r},t) = \sqrt{\gamma}\rho^{(\gamma-1)/2}$, where γ being the ratio of the specific heats, the sonic turbulent Mach number $\tilde{M}_s(\mathbf{r},t) = \sqrt{\langle|\mathbf{U}|^2\rangle}/\tilde{C}_s$, and the fluctuating Alfvénic speed $\tilde{V}_A = \tilde{B}/\sqrt{4\pi\tilde{\rho}}$, and the turbulent Alfvénic Mach number $\tilde{M}_A(\mathbf{r},t) = \sqrt{\langle|\mathbf{U}|^2\rangle}/\tilde{V}_A$. The turbulent Reynolds numbers and plasma beta $\tilde{\beta}$ can be defined correspondingly. We follow the evolution of these volume-integrated local quantities in time to understand the predominance of incompressible Alfvénic fluctuations in the Solar wind and local interstellar medium which may be responsible for turbulent cascades of energy. Furthermore, all the small-scale fluctuating parameters are measured in terms of their respective normalized quantities.

RESULTS AND DISCUSSION

We have developed a three dimensional compressible MHD code to numerically integrate Eqs. (1) to (4). The spatial discretization in our code uses a discrete Fourier representation of turbulent fluctuations based on a pseudospectral method, while the temporal integration is performed by Runge Kutta 4 method. The simulation parameters are described in Fig. (2). All the fluctuations are initialized isotropically (no mean fields are assumed) with random phases and amplitudes in Fourier space. This algorithm ensures conservation of total energy and mean fluid density per unit time in the absence of external random forcing. Our code is massively parallelized using Message Passing Interface (MPI) libraries to facilitate higher resolution. The initial isotropic turbulent spectrum of solenoidal as well as irrotational velocity components was chosen to be close to k^{-2}. It is to be noted, however, that our results of turbulent cascades do not depend upon the choice of initial spectrum. Therefore a flatter or steeper than k^{-2} spectrum leads qualitatively to similar results. The ISM turbulence code is evolved with time steps resolved self-consistently by the nonlinear interaction time scales associated with the convective plasma motion, i.e. $1/\mathbf{k}\cdot\mathbf{U}(\mathbf{k})$.

ISM plasma relaxes in the absence of external sources and sinks. As plasma evloves,

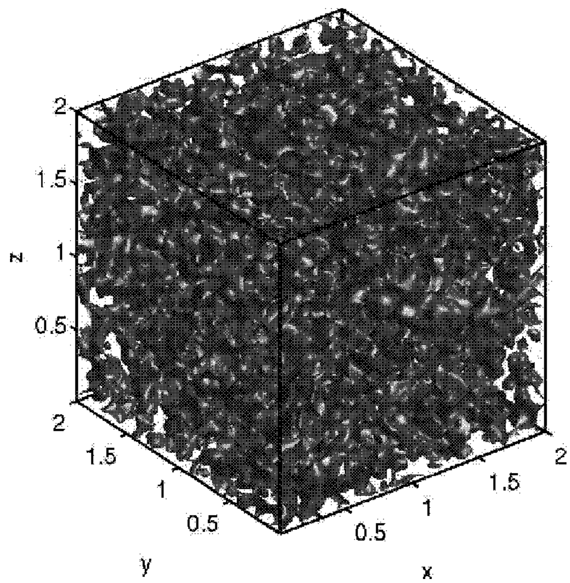

FIGURE 2. Snap-shot of turbulent magnetic field in three dimensions. Shown are the iso-surfaces of $|\mathbf{B}|$ turbulent fluctuations at intermediate time step. The computation domain has volume of $(2\pi)^3$, spectral resolution is $128^3 - 256^3$. Initial turbulent Reynolds numbers are $\tilde{R}_e = \tilde{R}_m \approx 200$ (drop to about 20-25%). Other parameters are $\gamma = 5/3, \beta_\circ = 10^{-3}, M_{A_\circ} = 1.-2., M_{s_\circ} = 5.-10., \eta = \mu = 10^{-4} - 10^{-5}, dt = 10^{-3}$. Different resolutions and initial conditions do not qualitatively change the evolution exhibited in the figure.

energy cascades amongst turbulent eddies of various scale sizes in the ISM magnetoplasma. A snapshot of the x-component of the fluctuating magnetic field in the ISM plasma is shown in Fig. (2). During the evolution, MHD turbulent fluctuations are dissipated nonlinearly gradually due to the finite Reynolds number in which nonlinear interaction transfer spectral energy from the smaller Fourier modes to the larger ones until turbulent excitation is terminated by Kolmogorov dissipation modes (k_d). This is where inertial range of the spectral cascades also terminates. The spectral migration of energy among various modes in the inertial range leads to a net decay of turbulent sonic Mach number M_s as shown in Fig. (**??**). The turbulent sonic Mach number continues to decay from a supersonic ($M_s > 1$) to a subsonic ($M_s < 1$) regime. This indicates that dissipative effects, triggered essentially by the nonlinear interactions, predominantly cause the supersonic MHD plasma fluctuations to damp strongly leaving primarily subsonic fluctuations in the MHD fluid. Furthermore, intrigued by the prediction of the NI theory, we monitor closely the evolution of the density fluctuations and turbulent sonic Mach number in view of understanding a relationship between the two, if there exists any. This is because the small amplitude and small-scale density fluctuations in the NI theory are shown to scale as $|\delta\rho| \sim \mathscr{O}(M_s^2) \sim \varepsilon$, where $\varepsilon = |\delta\rho|/\rho_0 \sim 10\%$ is treated as a small parameter in the perturbative expansion of ISM or solar wind velocity, magnetic field, pressure and density. Interestingly enough, we find a systematic development of the relationship between the density fluctuations and turbulent sonic Mach number. This is shown in Fig. (**??**) by solid-curve. Initially when the plasma is supersonic, the turbulent sonic Mach number is large enough compared to rms magnitude of the density fluctuations. The ratio of the density fluctuations and turbulent sonic Mach number in the initial

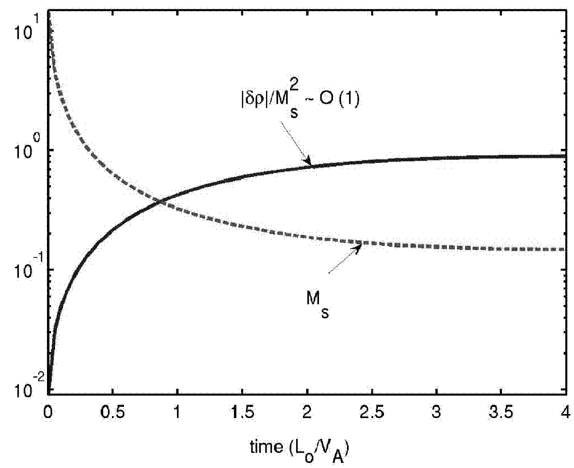

FIGURE 3. Snap-shot of turbulent magnetic field in three dimensions. Shown are the iso-surfaces of $|\mathbf{B}|$ turbulent fluctuations at intermediate time step.

stage thus shows a smaller value ($< 10^{-2}$). As the ISM plasma evolves, the nonlinear interactions lead to subsonic motions of turbulent modes which are characterized by a smaller magnitude of M_s i.e. $M_s < 1$. Since there are no damping processes involved in the evolution of the density fluctuations, they do not dissipate. By contrast, the density fluctuations begin to order quadratically with the turbulent Mach number. This consequently leads to the relationship $|\delta\rho|/(M_s^2) \sim \mathscr{O}(1)$ in our 3D simulations, as shown in Fig. (**??**) (the solid curve). Note that turbulent motion of the velocity field fluctuations associated with the smaller Mach number in the ISM is regarded predominantly as incompressible. This, in agreement with most observations [2, 3, 4, 5], points towards the fact that the ISM velocity field fluctuations are composed predominantly of (nearly) incompressible component. As a direct consequence of the ISM magnetoplasma being nearly incompressible, the density fluctations exhibit a weak compressibility in the gas and are convected predominantly passively in the background incompressible fluid flow field. This hypothesis can also be verified straightforwardly by investigating the density spectra which should be slaved to the incompressible velocity spectra [6].

The transition of compressible magnetoplasma from a(n) (initial) supersonic to a subsonic or nearly incompressible regime is gradual. This means the magnetofluid contains supersonic, and super Alfvénic modes initially in which highly compressible density fluctuations do not follow the velocity spectrum. It is the eventual decay of the turbulent Mach number to a subsonic regime that is responsible for the density fluctuations following the velocity fluctuations. In the subsonic regime, the compressibility weakens substantially so that the density fluctuations are advected only passively. A passively convected fluid follows a similar inertial range spectrum as that of its background flow field [8]. Likewise, subsonic density fluctuations are also expected to exhibit a spectrum similar to the background velocity fluctuations in the inertial range, as demonstrated in our work [6].

An alternate understanding of the passive scalar evolution of the density fluctuations associated essentially with incompressiblity can also be elucidated directly from the

continuity equation as follows.

$$(\partial_t + \mathbf{U} \cdot \nabla) \ln \rho = -(\nabla \cdot \mathbf{U}). \tag{5}$$

The rhs in Eq. (5) represents compressiblity of the velocity field fluctuations. It is clear that the density field is advected passively through the velocity field of the fluid when it is incompressible, i.e. when solenoidal component is vanishingly small. This leads to the expression of the solenoidal velocity field as $\nabla \cdot \mathbf{U} \simeq 0$, and fluid continuity equation becomes $(\partial_t + \mathbf{U} \cdot \nabla) \ln \rho \simeq 0$.

CONCLUSION

In conclusion, we have developed a self-consistent nonlinear 3D MHD fluid model to describe turbulent cascade processes that lead to a passively convected density fluctuation spectrum in the local interstellar medium and/or solar wind plasma. In an undriven situation, we find that an initial supersonic, compressible MHD plasma relaxes towards a state that comprises predominantly of subsonic and (nearly) incompressible plasma motion by virtue of suppressing the solenoidal velocity field fluctuations. The suppression is mediated explicitly by means of nonlinear interactions in which transverse Alfvénic fluctuations do not couple with co-existing longitudinal fast/slow modes. In view of the weak interaction between the two competing MHD modes, the nonlinear cascades are dominated by the Alfvénic fluctuations [7]. The latter is entirely responsible for generating the density fluctuations as passively convected structures or *pseudo modes* (i.e. non interacting modes). This has also been demonstrated by us in a recent 3D compressible MHD simulations of ISM plasma [7]. One of the important implications of the passive advection of the density fluctuations is that their characteristic spectrum can be determined from the background (nearly) incompressible velocity fluctuations. This strong correlation, is perhaps, responsible for the observed turbulent density spectrum in the local interstellar medium [2]. We finally point out that our results will not differ in the presence of external driving forces such as those arising from large-scale instability, supernovea and other possible sources as long as solenoidal component of the velocity field fluctuations is not affected dramatically to supersede the Alfvénic cascades.

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